

Контрольное домашнее задание 2. Дифференциальное исчисление

Рекомендуемое методическое пособие с теорией и с образцами решения задач контрольного домашнего задания:

А.В. Самохин, Л.Д. Жулёва, В.Н. Шевелёва, Ю.И. Дементьев, „Сборник задач по высшей математике. Часть 2. Пределы. Производные. Графики функций“, 2003 г. Библиотечный номер 536.

Данное пособие можно скачать в интернете по адресу
<http://vm.mstuca.ru/posobia/posobia.htm>

Либо только части, относящиеся к производным и приложениям производных,
по адресу

<http://vm.mstuca.ru/posobia/parts/proizv.pdf>,
<http://vm.mstuca.ru/posobia/parts/graph.pdf>

Задания 1 – 8. Найти производные функций.

Задание 9. Найти дифференциал функции.

Задание 10. Найти вторую производную функции.

Задание 11. Вычислить предел, применив правило Лопиталья.

Задания 12. Провести полное исследование функции и построить график. Написать уравнение касательной к графику в точке с абсциссой x_0 .

Задания 13. Провести полное исследование функции и построить график.

Задание 14. Найти частные производные $\frac{\partial z}{\partial x}$ и $\frac{\partial z}{\partial y}$ функции.

Задание 15. Найти $grad z$ в точке A и производную в точке A по направлению вектора \vec{a} .

Задание 16. Найти частные производные $\frac{\partial z}{\partial x}$ и $\frac{\partial z}{\partial y}$ от неявной функции.

Задание 17. Вычислить указанные производные, используя формулы производных сложной функции нескольких переменных.

Задание 18. Найти наибольшее и наименьшее значения функции z в замкнутой области D , ограниченной заданными линиями.

Вариант 1.

1. $y = e^x \arccos x$
2. $y = \frac{1 - \cos x}{2^x + 3}$
3. $y = \operatorname{arctg}(\ln x)$
4. $y = \sqrt{x} \ln(\sqrt{x} + \sqrt{x+3}) - \sqrt{x+3}$
5. $y = 2\sqrt{4x+3} - \frac{3}{\sqrt{x^2+1}}$
6. $y = \frac{\sin 3x}{\cos^2 x}$
7. $y = \frac{2}{3} \sqrt{(\operatorname{arctg} e^x)^3}$
8. $y = (\sin x)^{5e^x}$
9. $y = \arccos \sqrt{1-x}$
10. $y = \operatorname{tg}^2 x$
11. $\lim_{x \rightarrow 3} \frac{\log_3 x - 1}{\operatorname{tg} \pi x}$
12. $y = \frac{x^2}{4} (x^2 - 8); x_0 = 1$
13. $y = \frac{x^3 + 4}{x^2}$
14. $z = \frac{y}{x^2 - y^2}$
15. $z = x^2 + xy + y^2; A(1; 1); \vec{a} = \{2; -1\}$
16. $x^3 + 2y^3 + z^3 - 3xyz - 3y + 3 = 0$
17. $z = x^2 + xy + y^2, x = \sin t, y = \cos t; \frac{dz}{dt} = ?, dz = ?$
18. $z = 6xy - 9x^2 - 9y^2 + 4(x + y); D: x = 0, x = 1, y = 0, y = 2$

Вариант 2.

1. $y = \log_3 x \arcsin x$
2. $y = \frac{\sin x}{1 + \cos x}$
3. $y = \sqrt{x^3} \ln x + \frac{1}{x}$
4. $y = \sqrt{9x^2 - 12x + 5} \operatorname{arctg}(3x - 2)$
5. $y = 2\sqrt{x} - 4 \ln(2 + \sqrt{x})$
6. $y = (e^{\cos x} + 3)^4$
7. $y = 5^{x + \operatorname{arctg} x}$
8. $y = (\operatorname{ctg} x)^{2e^x}$
9. $y = 2^{\cos x}$
10. $y = x^2 e^x$
11. $\lim_{x \rightarrow 1} \frac{e^x - e}{\sin(x^2 - 1)}$
12. $y = 3x^4 - 4x^3; x_0 = 2$
13. $y = \frac{x^2 - x + 1}{x - 1}$
14. $z = x \arcsin(xy)$
15. $z = 2x^2 + 3xy + y^2; A(2; 1); \vec{a} = \{3; -4\}$
16. $z^2 + 3xz - 4yz - 9 = 0$
17. $z = \frac{1}{\sqrt{x^2 + y^2}}, x = u + v^2u, y = v - u^2v; \frac{\partial z}{\partial u} = ?, \frac{\partial z}{\partial v} = ?$
18. $z = 3x^2 + 3y^2 - 2x - 2y + 2; D: x = 0, y = 0, y = 1 - x$

Вариант 3.

1. $y = \sqrt{x^5} \left(1 - \frac{x}{2}\right)$

5. $y = \frac{1}{\sqrt{2}} \operatorname{arctg} \frac{4x+1}{\sqrt{2}}$

8. $y = (\operatorname{ctg} x)^{3^x}$

2. $y = \frac{x^3 - 3}{\operatorname{arctg} x}$

6. $y = \sqrt{\frac{1+x^2}{1-x}}$

9. $y = \ln^2 x^3$

3. $y = \cos^3 x \cdot 2^{\arcsin x}$

7. $y = \frac{1}{\operatorname{tg}^5 5x}$

10. $y = \ln(x^2 - 1)$

4. $y = 2x - \ln(1 + \sqrt{1 - e^{4x}})$

11. $\lim_{x \rightarrow \pi/2} \frac{1 - \sin^3 x}{\cos^2 x}$

12. $y = -\frac{(x^2 - 4)^2}{16}; x_0 = 1$

13. $y = \frac{2}{x^2 + 2x}$

14. $z = x^2 \sin \frac{x}{y}$

15. $z = \ln(x^2 + 3y^2); A(1; 1); \vec{a} = \{3; 2\}$

16. $2x^2 + 2y^2 + z^2 - 8xz - z + 8 = 0$

17. $z = x^2 y, y = \cos x; \frac{\partial z}{\partial x} = ?, \frac{\partial z}{\partial y} = ?, \frac{dz}{dx} = ?$

18. $z = 3x^2 + 3y^2 - x - y + 1; D: x = 1, x = 5, y = 0, x - y = 1$

Вариант 4.

1. $y = \frac{2}{3x^2} - \frac{x\sqrt[3]{x}}{2}$

5. $y = 3 \sin x \cos^2 x + \sin^3 x$

8. $y = (x^8 + 2)^{\frac{1}{x}}$

2. $y = \frac{1 + \cos x}{\arccos x}$

6. $y = \operatorname{arctg} \frac{1}{x}$

9. $y = 5^{\operatorname{tg} x}$

3. $y = \ln^3(1 + e^{3x})$

7. $y = \ln(e^x + \sqrt{1 + e^{2x}})$

10. $y = \cos^3 x$

4. $y = x \arcsin^2 x + 2\sqrt{1 - x^2} \arcsin x$

11. $\lim_{x \rightarrow 0} \frac{\sqrt{\cos x} - 1}{\sin^2 2x}$

12. $y = \frac{x^3}{27} (x - 4); x_0 = 1$

13. $y = \frac{4x^2}{x^2 + 3}$

14. $z = \frac{x}{x^2 + 3y^2}$

15. $z = \ln(5x^2 + 4y^2); A(1; 1); \vec{a} = \{2; -1\}$

16. $z^3 - 4y^2 x + 1 - 2z^2 = 0$

17. $z = \frac{x + y}{xy}, x = \operatorname{tg} t, y = \operatorname{ctg} t; \frac{dz}{dt} = ?$

18. $z = x^2 + 2xy - y^2 - 4x; D: y = x + 1, y = 0, x = 3$

Вариант 5.

1. $y = x^{10} \log_5 x$
2. $y = \frac{2^x}{\cos x + 5}$
3. $y = \frac{\sin^4 x}{\operatorname{ctg} x}$
4. $y = 2\sqrt{e^x + 1} + \ln \frac{\sqrt{e^x + 1} - 1}{\sqrt{e^x + 1} + 1}$
12. $y = \frac{x^2}{64} (32 - x^2); x_0 = 2$
15. $z = 5x^2 + 6xy; A(2; 1); \vec{a} = \{1; 2\}$
16. $z^3 + 3x^2z - 2xy = 0$
17. $z = e^{xy}, x = \sin t^2, y = \cos(1 - t); \frac{dz}{dt} = ?$
18. $z = 4(x - y) - x^2 - y^2; D: 2y = 4 - x, x = 0, 2y = x - 4$
5. $y = \frac{x^4}{81} \arcsin \frac{3}{x}$
6. $y = e^{-3x} \arcsin 2x$
7. $y = \ln(x + \sqrt{x^2 + 1})$
13. $y = \frac{12x}{x^2 + 9}$
8. $y = (\arcsin x)^{e^x}$
9. $y = \arccos 2x$
10. $y = \frac{\ln x}{x}$
11. $\lim_{x \rightarrow 10} \frac{\lg x - 1}{\sqrt{x - 9} - 1}$
14. $z = y^2 e^{x^2 + y^2}$

Вариант 6.

1. $y = \frac{2^x}{1 + 2^x}$
2. $y = 3^{\arcsin^2 x}$
3. $y = \lg \ln \operatorname{ctg} x$
4. $y = \frac{x^4}{81} \arcsin \frac{3}{x} + \sqrt{x^2 - 9}$
12. $y = \frac{x^3}{16} (8 - 3x); x_0 = 1$
15. $z = \arctg(xy^2); A(2; 3); \vec{a} = \{4; -3\}$
16. $zx^2 + y^2 - 4z^3y - 1 = 0$
17. $z = \frac{x}{\sqrt{x^2 - y}}, x = \sin t, y = t \cos t; \frac{dz}{dt} = ?$
18. $z = x^3 + y^3 - 9xy + 27; D: x = 0, x = 3, y = 0, y = 3$
5. $y = \ln \arcsin \sqrt{1 - e^{2x}}$
6. $y = (1 + \sqrt[4]{x^3}) \arcsin x$
7. $y = \frac{\cos x}{\sin^2 x} + \ln(\sin x)$
13. $y = \frac{x^2 - 3x + 3}{x - 1}$
8. $y = (x)^{e^{\cos x}}$
9. $y = x^3 e^x$
10. $y = x \operatorname{arctg} x$
11. $\lim_{x \rightarrow 3} \frac{\sqrt[3]{5 + x} - 2}{\sin \pi x}$
14. $z = \sqrt[3]{4xy + y^2}$

Вариант 7.

1. $y = \sqrt[7]{x^3} \sin x$
5. $y = \frac{\sqrt{1 - x^2}}{x} + \arcsin x$
8. $y = (\sin 2x)^{\frac{\ln \cos x}{4}}$

2. $y = \frac{4 + x^3}{x - \operatorname{ctg} x}$ 6. $y = e^{2x} \ln(1 + x^2)$ 9. $y = \cos^2 \frac{1}{x^2}$
 3. $y = \operatorname{tg}^2 x + \frac{1}{\cos x}$ 7. $y = \sqrt{2x - x^2} + \frac{1}{3x^3}$ 10. $y = \ln(12 + x)$
 4. $y = \frac{1}{2} \ln(e^{2x} + 1) - 2 \operatorname{arctg} e^x$ 11. $\lim_{x \rightarrow \pi} \frac{e^\pi - e^x}{\sin 5x - \sin 3x}$
 12. $y = \frac{1}{9} (x^2 - 3)^2; x_0 = 2$ 13. $y = \frac{4 - x^3}{x^2}$ 14. $z = e^{xy} (2x + y^2)$
 15. $z = \arcsin \frac{x^2}{y}; A(1; 2); \vec{a} = \{5; -12\}$
 16. $2x^3 - 5x + z^3 + y^3 - 3xyz + 8 = 0$
 17. $z = x^2y + xy^2, x = u + v, y = u - v; \frac{\partial z}{\partial u} = ?, \frac{\partial z}{\partial v} = ?$
 18. $z = x^2 + 2xy - y^2 - 2(x - y); D: y = x + 2, y = 0, x = 2$

Вариант 8.

1. $y = \log_5 x \arccos x$ 5. $y = \operatorname{arctg} \frac{1}{1 - \sqrt{x}}$ 8. $y = (\cos x)^{\frac{2 \ln \cos x}{5}}$
 2. $y = \frac{e^x}{1 - x^2}$ 6. $y = \left(x^5 + 3x + \frac{1}{x}\right)^{10}$ 9. $y = \frac{1}{3^x}$
 3. $y = 3 \sin 2x \cos^2 x$ 7. $y = \sqrt{\ln(x^2 + 1)}$ 10. $y = \cos^2 3x$
 4. $y = -\frac{1}{3 \sin^2 x} + \frac{1}{2} \ln \frac{1 + \sin x}{1 - \sin x}$ 11. $\lim_{x \rightarrow 0} \frac{1 - \sqrt{\cos x}}{1 - \cos \sqrt{x}}$
 12. $y = \frac{x^2}{27} (x^2 - 18); x_0 = 1$ 13. $y = \frac{x^2 - 4x + 1}{x - 4}$ 14. $z = (x^2 - y^2) \cos xy$
 15. $z = \ln(3x^2 + 4y^2); A(1; 3); \vec{a} = \{2; -1\}$
 16. $xy + xz + yz^2 - 1 = 0$
 17. $z = \ln(e^x + e^y), y = x^2; \frac{\partial z}{\partial x} = ?, \frac{dz}{dx} = ?$
 18. $z = x^2 - xy + 2y^2 + 3x + 2y + 1; D: x = 0, y = 0, x + y + 5 = 0$

Вариант 9.

1. $y = \sqrt[5]{x} 3^x$ 5. $y = \frac{2}{3} \sqrt{\frac{x-1}{x+1}}$ 8. $y = (\operatorname{tg} x)^{4e^x}$
 2. $y = \frac{x^2 + 5x - 6}{\ln x}$ 6. $y = \frac{1}{2 \sin^2 x} + \ln(\operatorname{tg} x)$ 9. $y = e^{-\frac{1}{x^2}}$
 3. $y = e^{\frac{1}{\cos x}}$ 7. $y = \operatorname{arctg} \frac{2 \operatorname{tg} x}{1 - \operatorname{tg} x}$ 10. $y = 5\sqrt{x}$

4. $y = \ln(e^{5x} + \sqrt{e^{10x} - 1})$ 11. $\lim_{x \rightarrow 0} \frac{\sqrt{x+2} - \sqrt{2}}{\sin 3x}$
 12. $y = \frac{1}{8}(3x^5 - 5x^3); x_0 = 2$ 13. $y = \frac{2x^3 + 1}{x^2}$ 14. $z = y \ln(x^2 - y^2)$
 15. $z = 3x^4 + 2x^2y^3; A(-1; 2); \vec{a} = \{4; -3\}$
 16. $x^3 + y^3 + z^3 - 3xyz = 0$
 17. $z = x^2y + \frac{y}{x}, x = e^t, y = \ln(t^2 + t); \frac{dz}{dt} = ?$
 18. $z = x^2 - xy + y^2; D: |x| + |y| = 1$

Вариант 10.

1. $y = 2x 5^x$ 5. $y = \frac{2 \cos x}{\sin^4 x} + \frac{3 \cos x}{\sin^2 x}$ 8. $y = (\cos 5x)^{e^x}$
 2. $y = \frac{1 + \cos x}{1 - \cos x}$ 6. $y = (x^3 + 3x^4) \log_3 x$ 9. $y = \operatorname{arctg} x^2$
 3. $y = \operatorname{ctg}^3 x - \frac{1}{\sin x}$ 7. $y = \frac{\ln(x^2 + 2x)}{3x}$ 10. $y = xe^x$
 4. $y = \frac{x^3}{3} \arccos x - \frac{2 + x^2}{9} \sqrt{1 - x^2}$ 11. $\lim_{x \rightarrow 1} \frac{2^x - 2}{\ln x}$
 12. $y = \frac{x^4}{64}(x - 5); x_0 = 2$ 13. $y = \frac{(x - 1)^2}{x^2}$ 14. $z = \operatorname{arctg} \frac{y}{x}$
 15. $z = 3x^2y^2 + 5y^2x; A(1; 1); \vec{a} = \{2; 1\}$
 16. $5xy^3 + 4z^2y - x^3 + z^3 - 6 = 0$
 17. $z = \frac{1 + xy}{x + y}, x = \sin(uv), y = \cos(uv); \frac{\partial z}{\partial u} = ?, \frac{\partial z}{\partial v} = ?$
 18. $z = x^2 + y^2 - 12x + 16y; D: x^2 + y^2 = 25$

Вариант 11.

1. $y = \frac{1}{6(1 - x^2)}$ 5. $y = \ln \frac{\sqrt{2} + \operatorname{tg} x}{\sqrt{2} - \operatorname{tg} x}$ 8. $y = (\operatorname{tg} x)^{4e^x}$
 2. $y = 3\sqrt[3]{x} \arccos x$ 6. $y = \frac{1}{\ln^2 x}$ 9. $y = \frac{1}{3x + 1}$
 3. $y = \sin^6 x + \cos^6 x$ 7. $y = \frac{4}{3} \sqrt[4]{\frac{x - 1}{x + 2}}$ 10. $y = 2^{\sin x}$
 4. $y = (2x + 3)^4 \arcsin \frac{1}{2x + 3}$ 11. $\lim_{x \rightarrow 0} \frac{e^{\sin 2x} - e^{\sin x}}{\operatorname{tg} x}$
 12. $y = x^4 - 8x^3 + 16x^2; x_0 = 1$ 13. $y = \frac{x^2}{(x - 1)^2}$ 14. $z = \arcsin \frac{x^2}{y}$

15. $z = \operatorname{arctg} \frac{y}{x}; A(1; 1); \vec{a} = \{3; -4\}$

16. $\ln z = x^2 + yz - 1$

17. $z = \frac{1}{2} \ln \frac{x}{y}, x = \operatorname{tg}^2 t, y = \operatorname{ctg}^3 t; \frac{dz}{dt} = ?$

18. $z = x^2 + xy - 2; D : y = 4x^2 - 4, y = 0$

Вариант 12.

1. $y = \frac{4}{x^3} + 5\sqrt[5]{x^4} + 2$

5. $y = \operatorname{arctg} \frac{1}{x} + \frac{\sqrt{x^2 - 1}}{x}$

8. $y = (x - 5)^{\operatorname{ctg} 2x}$

2. $y = \frac{\operatorname{arctg} x}{1 + x^2}$

6. $y = e^{\sin x} \cos 2x$

9. $y = \log_{\frac{1}{3}}(1 - x)$

3. $y = e^{\sqrt{1 + \ln x}}$

7. $y = x + \ln \sqrt{\frac{1 + x}{1 - x}}$

10. $y = \arcsin x$

4. $y = \sqrt{1 - x^2} + \arcsin x$

11. $\lim_{x \rightarrow 1} \frac{1 - x}{\log_2 x}$

12. $y = \frac{3}{2} (x^4 - 2x^2); x_0 = 2$

13. $y = \left(1 + \frac{1}{x}\right)^2$

14. $z = x \ln(3x^2 + 2y^3)$

15. $z = \ln(8x^2 + 3y); A(1; 4); \vec{a} = \{2; -1\}$

16. $\ln(z + xy) = z^2 - y$

17. $z = \arcsin \frac{x}{y}, y = \sqrt{x^2 + 1}; \frac{\partial z}{\partial x} = ?, \frac{dz}{dx} = ?$

18. $z = x^2 + 2xy - 10; D : y = x^2 - 4, y = 0$

Вариант 13.

1. $y = \frac{1 + x^2}{2} \operatorname{arctg} x$

5. $y = \cos^2 x - 2 \ln \cos x$

8. $y = (x^3 + 4)^{\operatorname{tg} x}$

2. $y = \frac{1 + \sqrt{x}}{1 - \sqrt{x}}$

6. $y = \ln \sin \frac{2x + 4}{x + 1}$

9. $y = \log_2(2x - 1)$

3. $y = \frac{2}{3} \sqrt{(1 + \ln x)^3}$

7. $y = \frac{4 + x^4}{x^3} \operatorname{arctg} \frac{x}{2}$

10. $y = 2^{x^2}$

4. $y = e^{2x} (2 \sin 2x - \cos 2x)$

11. $\lim_{x \rightarrow 3} \frac{2^x - 8}{x - 3}$

12. $y = x^2 (x - 2)^2; x_0 = -1$

13. $y = \frac{3 - x^2}{x + 2}$

14. $z = \arccos \frac{x}{\sqrt{y}}$

15. $z = \arcsin \frac{x}{x + y}; A(1; 1); \vec{a} = \{4; 3\}$

16. $x \cos y + y \cos z + z \cos x = 1$

$$17. z = x^2 \ln y, \quad x = \frac{u}{v}, \quad y = 3u - 2v; \quad \frac{\partial z}{\partial u} = ?, \quad \frac{\partial z}{\partial v} = ?$$

$$18. z = x^2 - y^2 + 18; \quad D : x^2 + y^2 = 9$$

Вариант 14.

$$1. y = x^2 \sqrt{x} + \frac{1}{4x^2} - 6x \quad 5. y = \ln \operatorname{tg} \frac{x}{2} - \frac{x}{\sin x}$$

$$8. y = x^{\sin^3 x}$$

$$2. y = \frac{\ln x}{\sin x} + x 2^x \quad 6. y = 2^{\sin x}$$

$$9. y = \sqrt[5]{1 - x^3}$$

$$3. y = \frac{1}{3} \operatorname{arctg} \frac{x}{3} + e^{\sin x} \quad 7. y = \log_{16} \log_5 \operatorname{ctg} x$$

$$10. y = (1 + x^2) \operatorname{arctg} x$$

$$4. y = (1 + \ln \sin 2x)^2$$

$$11. \lim_{x \rightarrow \pi/4} \frac{\sin x - \cos x}{\ln \operatorname{tg} x}$$

$$12. y = \frac{x^3}{9} (x + 4); \quad x_0 = 3 \quad 13. y = \frac{x^2 + 6x + 3}{x + 4}$$

$$14. z = \frac{4x}{x^3 - y^3}$$

$$15. z = \operatorname{arctg}(xy); \quad A(1; 1); \quad \vec{a} = \{1; 1\}$$

$$16. xyz = e^{-(x+y+z)}$$

$$17. z = x \sin y + y \cos x, \quad x = \frac{u}{v}, \quad y = u^3 v^2; \quad dz = ?$$

$$18. z = 2x^3 + 4x^2 + y^2 - 2xy; \quad D : y = x^2, \quad y = 4$$

Вариант 15.

$$1. y = 3\sqrt[3]{x} - 2\sqrt{x^3} + 4 \quad 5. y = 5e^{-x^2} + \frac{1}{\operatorname{arctg} x}$$

$$8. y = (x^2 - 1)^{\ln x}$$

$$2. y = \frac{x}{4} (\operatorname{tg} x - \operatorname{ctg} x) \quad 6. y = \sqrt[5]{\frac{x^2 - 1}{x^2 + 2}}$$

$$9. y = \sqrt[3]{1 - x^3}$$

$$3. y = \frac{2}{3} \sqrt{(1 + \ln x)^3} \quad 7. y = \log_4 \log_2 \operatorname{tg} x$$

$$10. y = \operatorname{ctg} 3x$$

$$4. y = \frac{\arcsin x}{\sqrt{1 - x^2}} + \frac{1}{2} \ln \frac{1 - x}{1 + x}$$

$$11. \lim_{x \rightarrow 1} \frac{1 - x^2}{\sin \pi x}$$

$$12. y = \frac{x^3}{72} (x - 8); \quad x_0 = 2 \quad 13. y = \frac{-8x}{x^2 + 4}$$

$$14. z = 12 \cos^2 \left(\frac{x}{3} - \frac{y}{4} \right)$$

$$15. z = x - 3y + \sqrt{3xy}; \quad A(3; 4); \quad \vec{a} = \{1; -2\}$$

$$16. \ln(x + y + z) = \sin(x + y + z)$$

$$17. z = \ln(x^2 + y^2), \quad x = u^2 + v, \quad y = v^2 - u; \quad \frac{\partial z}{\partial u} = ?, \quad \frac{\partial z}{\partial v} = ?$$

$$18. z = y^2 + xy - 2; \quad D : y = 0, \quad y = 4x^2 - 4$$

Вариант 16.

1. $y = \frac{\sqrt{x}}{1 + \sqrt{x}}$

5. $y = \arccos \frac{1}{x^2}$

8. $y = (x^4 + 5)^{\operatorname{ctg} x}$

2. $y = \left(x^2 + \frac{1}{x}\right) \log_3 x$

6. $y = e^{3x} \cos 2x$

9. $y = \frac{x^2}{1 - x}$

3. $y = 2^{x^2} - e^{-x^2}$

7. $y = \sqrt{\operatorname{ctg} x} + \frac{1}{3} \sqrt{\operatorname{tg}^3 x}$

10. $y = \operatorname{arctg}(-x)$

4. $y = \frac{x}{2} (\cos \ln x + \sin \ln x)$

11. $\lim_{x \rightarrow \pi/3} \frac{1 - 2 \cos x}{\sin(\pi - 3x)}$

12. $y = (x + 1)^2 (x - 1)^2; x_0 = 2$

13. $y = \frac{1}{x^2 - 1}$

14. $z = (2x + y) e^{-xy}$

15. $z = \sqrt{4 + x^2 + y^2}; A(2; 1); \vec{a} = \{1; -1\}$

16. $x + y + z = e^{-(x+y+z)}$

17. $z = e^{x^2 y}, x = uv, y = \frac{v}{u}; \frac{\partial z}{\partial u} = ?, \frac{\partial z}{\partial v} = ?$

18. $z = x^2 - 2xy + y^2; D : x + y = 1, y = 0, x = 0$

Вариант 17.

1. $y = \frac{3e^x}{\sqrt{x}}$

5. $y = \frac{3}{2} \sin^2 x + \ln(\operatorname{tg} x)$

8. $y = (\sin x)^{5^x}$

2. $y = \frac{1}{2} \operatorname{tg} x \operatorname{arctg} x$

6. $y = \sqrt{3} \operatorname{arctg} \frac{x}{\sqrt{3}}$

9. $y = \sqrt{\operatorname{tg} x}$

3. $y = \ln \cos \frac{2x + 3}{2x + 1}$

7. $y = 3^{\operatorname{ctg}^2 x}$

10. $y = (5 - 2x)^6$

4. $y = \arcsin e^{-2x} + \ln \sqrt{e^{4x} - 1}$

11. $\lim_{x \rightarrow -1} \frac{x^3 + 1}{\sin(x + 1)}$

12. $y = \frac{1}{8} x^2 (x - 4)^2; x_0 = -2$

13. $y = \frac{3x^4 + 1}{x^3}$

14. $z = \sqrt{x} \sin \frac{y}{x}$

15. $z = \arcsin xx + y; A(3; 4); \vec{a} = \{-2; 1\}$

16. $z \ln(x + z) = \frac{xy}{z}$

17. $z = x + \ln(x^2 + y^2), x = t^2, y = t(t + 1); \frac{dz}{dt} = ?$

18. $z = y^2 - x^2 + 8; D : x^2 + y^2 = 4$

Вариант 18.

1. $y = \frac{2}{x^2 - 4x + 5}$

5. $y = x \arccos x - \sqrt{1 - x^2}$

8. $y = x^{e^{\operatorname{ctg} x}}$

2. $y = x\sqrt{x} \operatorname{arctg} x$ 6. $y = \sqrt[3]{\sin^2 x} + \frac{1}{\cos x}$ 9. $y = \sin^3 2x$
3. $y = e^{\operatorname{tg} 2x} \ln(1 - x^3)$ 7. $y = \ln \operatorname{arctg} \sqrt{e^{4x} - 1}$ 10. $y = \operatorname{tg} 5x$
4. $y = 3 \arcsin \frac{3}{x+2} + \sqrt{x^2 + 4x - 5}$ 11. $\lim_{x \rightarrow 2} \frac{\operatorname{tg} x - \operatorname{tg} 2}{\ln x - \ln 2}$
12. $y = \frac{27}{32} x^2 (2 - x); x_0 = 2$ 13. $y = \frac{3x - 2}{x^3}$ 14. $z = \operatorname{arctg} \frac{x^2}{y}$
15. $z = \sqrt{x^2 + y^2}; A(4; 3); \vec{a} = \{-1; 2\}$
16. $x \sin y + y \sin z + z \sin x = 3$
17. $z = (x^2 + y^2)e^{x^2 - y^2}, x = t + 1, y = \frac{1}{t^2}; \frac{dz}{dt} = ?$
18. $z = x^2 + y^2 - 6x + 4y + 2; D: x = 1, x = 4, y = -3, y = 2$

Вариант 19.

1. $y = \frac{\ln x}{2x + 1}$ 5. $y = (1 - 2x^2) \operatorname{arctg} x$ 8. $y = (\sqrt{x})^{2x}$
2. $y = \sqrt[3]{\operatorname{tg} 3x}$ 6. $y = x \sqrt{\frac{2x + 1}{2x - 1}}$ 9. $y = \sin \sqrt{x}$
3. $y = \ln^2 x - \ln \ln x$ 7. $y = \log_3 \frac{1}{1 - x^4}$ 10. $y = e^x \cos x$
4. $y = \arcsin e^x - \sqrt{1 - e^{2x}}$ 11. $\lim_{x \rightarrow 0} \frac{e^x + e^{-x} - 2}{\sin^2 x}$
12. $y = 3x^4 + 4x^3; x_0 = 1$ 13. $y = \frac{x^3 - 32}{x^2}$ 14. $z = \sqrt{xy + 2y^2}$
15. $z = 5x^2 y + y^2 x; A(1; 1); \vec{a} = \{-2; -1\}$
16. $xe^y + ye^z + ze^x = 2$
17. $z = \ln(x^2 + y^2), x = uv, y = \frac{u}{v - u}; \frac{\partial z}{\partial u} = ?, \frac{\partial z}{\partial v} = ?$
18. $z = y^2 + 2xy - x^2; D: y = x - 1, x = 0, y = 3$

Вариант 20.

1. $y = \frac{2x + 1}{4 - x^2}$ 5. $y = 3 \sin x \cos^2 x + \sin^3 x$ 8. $y = (\cos 2x)^{\cos 2x}$
2. $y = \sqrt[3]{\operatorname{arctg}^2 x}$ 6. $y = \ln \sqrt[5]{e^{5x} - e^{-5x}}$ 9. $y = \log_3 (x^2 - 1)$
3. $y = 5^x (\operatorname{tg} x + \operatorname{ctg} x)$ 7. $y = e^{5x} - \frac{1}{2} \operatorname{tg} 4x + \frac{1}{4} x^4$ 10. $y = (7x - 3x^2)^5$
4. $y = \frac{1}{\sqrt{2}} \ln \left(\sqrt{2} \operatorname{tg} x + \sqrt{1 + 2 \operatorname{tg}^2 x} \right)$ 11. $\lim_{x \rightarrow 0} \frac{2^{3x} - 3^{5x}}{\sin 7x - 2x}$

$$12. y = \frac{x^3(x^2 - 15)}{81}; x_0 = 1 \quad 13. y = \frac{1 - 2x^2}{x^2} \quad 14. z = \ln \operatorname{tg} \left(\frac{x}{3} - \frac{y}{6} \right)$$

$$15. z = \operatorname{arctg} \frac{y}{x}; A(2; 2); \vec{a} = \{-3; 1\}$$

$$16. \operatorname{tg}(x + z) = ye^z$$

$$17. z = \frac{x - 2y}{x + 2y}, x = \frac{u - v}{u}, y = \frac{v}{u + v}; \frac{\partial z}{\partial u} = ?, \frac{\partial z}{\partial v} = ?$$

$$18. z = y^2 + 2xy - x^2 + 2(x - y); D: y = x - 2, x = 0, y = 2$$

Вариант 21.

$$1. y = \frac{2}{x^3} + \frac{6}{\sqrt{x}} + \frac{3x^4}{4} \quad 5. y = x \operatorname{arctg} x + \ln \sqrt{1 + x^2} \quad 8. y = (\sin \sqrt{x})^{e^x}$$

$$2. y = \frac{\cos x}{1 + \sin x} \quad 6. y = \operatorname{arctg} \frac{2x}{1 - x^2} \quad 9. y = 3e^{\operatorname{arctg} x}$$

$$3. y = \cos^3 x 2^{\operatorname{tg} x} \quad 7. y = 2^{\operatorname{ctg} \frac{1}{x}} \quad 10. y = x \ln x$$

$$4. y = \sqrt{1 + x^2} \operatorname{arctg} x - \ln(x + \sqrt{1 + x^2}) \quad 11. \lim_{x \rightarrow 0} \frac{e^{2x} - e^x}{x + \operatorname{tg} x^2}$$

$$12. y = \frac{x^3}{9}(4 - x); x_0 = 1 \quad 13. y = \frac{4}{x^2 + 2x - 3} \quad 14. z = 2 \arcsin(x\sqrt{y})$$

$$15. z = \arcsin \frac{x}{y}; A(3; 5); \vec{a} = \{1; \sqrt{3}\}$$

$$16. z = x + \operatorname{arctg} \frac{y}{z - x}$$

$$17. z = \frac{x^2 + 2y^3}{x + 2y}, x = \frac{u + v^2}{u}, y = \frac{v^2}{u + v}; \frac{\partial z}{\partial u} = ?, \frac{\partial z}{\partial v} = ?$$

$$18. z = 2x^2 + xy + y^2 - 2x + 3y + 1; D: x = 0, y = 0, y = x - 5$$

Вариант 22.

$$1. y = \frac{1 - 10^x}{1 + 10^x} \quad 5. y = \sqrt{\ln x + 1} + \ln(\sqrt{x} + 1) \quad 8. y = (x^2 + 1)^{\cos x}$$

$$2. y = 2x \arcsin x \quad 6. y = 2e^{2x} \sin x \cos x \quad 9. y = \cos \left(2 - \frac{1}{x} \right)$$

$$3. y = \lg \ln \operatorname{ctg} x \quad 7. y = \frac{\operatorname{arctg} \sqrt{x} - \sqrt{x}}{x} \quad 10. y = \frac{1}{2} \log_2(3 + x^2)$$

$$4. y = \operatorname{arctg} \sqrt{x^2 - 1} - \frac{\ln x}{\sqrt{x^2 - 1}} \quad 11. \lim_{x \rightarrow 0} \frac{e^{4x} - e^{2x}}{2 \operatorname{tg} x - \sin x}$$

$$12. y = \frac{x^3}{27}(15 - x^2); x_0 = 1 \quad 13. y = - \left(\frac{x}{x + 2} \right)^2 \quad 14. z = \sin^2(4x + y)$$

$$15. z = \ln(2x^2 + 4y^2); A(2; 1); \vec{a} = \{-2; -1\}$$

$$16. z = \sqrt{x^2 - y} \cdot \operatorname{tg} \frac{z}{\sqrt{y^2 - x}}$$

$$17. z = x^3 \sin y + y^3 \cos x, x = t(t - 1), y = \frac{t^2}{t - 1}; \frac{dz}{dt} = ?$$

$$18. z = 4x^2 + 2xy + y^2; D : 2|x| + |y| = 1$$

Вариант 23.

- | | | |
|---|--|------------------------------|
| 1. $y = 3^x \operatorname{tg} x$ | 5. $y = (2x - 1)^4 \arcsin \frac{1}{2x - 1}$ | 8. $y = x^{\arcsin x}$ |
| 2. $y = \frac{2 - x}{x^2 + \sqrt{x}}$ | 6. $y = e^{\operatorname{arctg} \sqrt{x}}$ | 9. $y = x^2 \sqrt[3]{\ln x}$ |
| 3. $y = (3 + 2x^2)^5$ | 7. $y = \sqrt[3]{\operatorname{ctg} \frac{1}{x}}$ | 10. $y = \cos^2 2x$ |
| 4. $y = \operatorname{arctg} \sqrt{x^2 - 1} - \frac{\ln x}{\sqrt{x^2 - 1}}$ | 11. $\lim_{x \rightarrow 1} \frac{\cos \frac{\pi x}{2}}{1 - \sqrt{x}}$ | |
| 12. $y = \frac{16}{27} (x + 1) (1 - x)^3; x_0 = -1$ | 13. $y = \frac{4x}{(x + 1)^2}$ | 14. $z = x^2 e^{x^2 - y^2}$ |
| 15. $z = \ln(x^2 + 4y^2); A(6; 4); \vec{a} = \{-1; \sqrt{3}\}$ | | |
| 16. $\frac{x}{z} = \ln \frac{z}{y} + 1$ | | |
| 17. $z = x \ln(xy), x = u^2 v, y = \frac{v^2}{u}; \frac{\partial z}{\partial u} = ?, \frac{\partial z}{\partial v} = ?$ | | |
| 18. $z = x^2 + 12y + y^2 - 16x; D : x^2 + y^2 = 25$ | | |

Вариант 24.

- | | | |
|---|---|------------------------------|
| 1. $y = \sqrt[3]{x^2} \cos x$ | 5. $y = \arcsin(1 - x) + \sqrt{2x - x^2}$ | 8. $y = (3 \sin x)^{\cos x}$ |
| 2. $y = \frac{x + e^x}{x - e^x}$ | 6. $y = 3 \operatorname{arctg} \frac{x}{3} + e^{\arcsin x}$ | 9. $y = 7 \sin^2 x$ |
| 3. $y = 3^{\sin \frac{1}{x}}$ | 7. $y = \frac{\sqrt{1 - x^2}}{x}$ | 10. $y = \sqrt{1 - x^2}$ |
| 4. $y = \ln(3x - 2 + \sqrt{9x^2 - 12x + 5})$ | 11. $\lim_{x \rightarrow \pi} \frac{1 - \sin \frac{x}{2}}{\pi - x}$ | |
| 12. $y = \frac{x^3}{36} (x + 8); x_0 = 1$ | 13. $y = \frac{x^2 - 1}{x^2 + 1}$ | 14. $z = y e^{\frac{y}{x}}$ |
| 15. $z = 3x^2 - 4xy + 5y^2; A(-1; -1); \vec{a} = \{-3; 2\}$ | | |
| 16. $e^{z/x} \cos \frac{x}{y} = \frac{x}{y}$ | | |
| 17. $z = \operatorname{arctg} \frac{y}{x}, x = e^{2t} + 1, y = e^{2t} - 1; \frac{dz}{dt} = ?$ | | |
| 18. $z = y^2 - 2xy - 2; D : y = 0, y = 16x^2 - 4$ | | |

Вариант 25.

1. $y = \frac{1 + e^x}{1 - e^x}$

5. $y = -\frac{1}{2 \sin^2 x} + \ln(\arcsin x)$

8. $y = (\operatorname{arctg} 2x)^x$

2. $y = \operatorname{arctg} x \log_3 x$

6. $y = \frac{\sqrt{1-x^2}}{x} + \arcsin x$

9. $y = (2x + 1)^{15}$

3. $y = \sqrt{\frac{2x+1}{x}}$

7. $y = \frac{1}{(2x-5)^5} + \frac{2}{\sqrt{x}}$

10. $y = e^{-x^2}$

4. $y = \operatorname{arctg}(\ln x) + \ln(\operatorname{arctg} x)$

11. $\lim_{x \rightarrow \pi/4} \frac{\ln \operatorname{tg} x}{\cos 2x}$

12. $y = \frac{x^4 - 8x^2 - 9}{5}; x_0 = 1$

13. $y = \frac{x^3}{x^2 - 1}$

14. $z = \left(xy + \frac{x}{y}\right)^2$

15. $z = 5x^2 + 2xy + 3y^2; A(-3; 2); \vec{a} = \{-2; 4\}$

16. $ze^z - x \ln y = \sqrt{\frac{x}{y}}$

17. $z = \frac{x^2}{y}, x = u - 2v, y = v + 2u; \frac{\partial z}{\partial u} = ?, \frac{\partial z}{\partial v} = ?$

18. $z = x^2 + 2xy + y^2; D : y = x + 1, x = 0, y = 0$