

Контрольное домашнее задание 1.
Двойные интегралы. Дифференциальные уравнения.

Срок сдачи – до 28 октября включительно
Одна защищённая задача оценивается в 1 балл
Максимальное количество баллов за КДЗ-1 равно 14

Рекомендуемые методические пособия с теорией и с образцами решения задач
контрольного домашнего задания:

Методическое пособие, относящееся к кратным интегралам:
<http://vm.mstuca.ru/posobia/parts/krint.pdf>

Методическое пособие, относящееся к дифференциальным уравнениям:
*Л. Д. Жулёва, А. В. Самохин, В. Н. Шевелёва, Ю. И. Дементьев „Сборник
задач по высшей математике. Часть 4. Интегралы. Дифференциальные
уравнения“, 2005 г. Библиотечный номер 1448.*

Данное пособие можно скачать в интернете по адресу:
<http://vm.mstuca.ru/posobia/posobia.htm>

Либо только часть, относящуюся к дифференциальному исчислению, по адресу:
<http://vm.mstuca.ru/posobia/parts/diffur.pdf>

Задание 1. Поменять порядок интегрирования.

Задания 2 – 3. Вычислить двойные интегралы.

Задание 4. Найти площадь фигуры, ограниченной данными линиями.

Задание 5. Пластинка D задана ограничивающими её кривыми, μ — поверхностная плотность. Найти массу пластинки.

Задания 6 – 7. Вычислить криволинейные интегралы вдоль линии L от точки M до точки N .

Задание 8. Для данного дифференциального уравнения методом изоклин построить интегральную кривую, проходящую через точку M .

Задания 9 – 14. Решить дифференциальные уравнения.

Вариант 1.

$$1. \int_{-2}^{-1} dy \int_0^{\sqrt{2+y}} f(x, y) dx + \int_{-1}^0 dy \int_0^{\sqrt{-y}} f(x, y) dx$$

$$2. \iint_D (12x^2y^2 + 16x^3y^3) dx dy; \quad D: x = 1, y = x^2, y = -\sqrt{x}$$

$$3. \iint_D ye^{\frac{xy}{2}} dx dy; \quad D: y = \ln 2, y = \ln 3, x = 2, x = 4$$

$$4. y^2 - 2y + x^2 = 0, y^2 - 4y + x^2 = 0, y = \frac{x}{\sqrt{3}}, y = \sqrt{3}x$$

$$5. D: x^2 + y^2 = 4, x^2 + y^2 = 9, y = -x, y = 0 (x \leq 0, y \geq 0), \mu = \frac{y - 4x}{x^2 + y^2}$$

$$6. \int_L (x + y) dx - (x - y) dy, L: \text{отрезок } MN, M(2; 0), N(4; 5)$$

$$7. \int_L (x^2y - 3x) dx + (y^2x + 2y) dy,$$

$$L: x = 3 \cos t, y = 3 \sin t, (y \geq 0), M(3; 0), N(-3; 0)$$

$$8. y' = y - x^2, M(1; 2)$$

$$9. 6x dx - 2y dy = 2y x^2 dy - 3xy^2 dx$$

$$10. y' + y \operatorname{tg} x = \cos^2 x, y(\pi) = 0$$

$$11. (1 + x^2) y'' + 2x y' = 2x$$

$$12. y''' + 8y'' + 15y' = 0$$

$$13. y'' - 2y' + y = 9e^{-2x}$$

$$14. y'' + y' = 16x + 10, y(0) = 0, y'(0) = 0$$

Вариант 2.

1. $\int_0^1 dy \int_{2y^2}^{3-y} f(x, y) dx$

2. $\iint_D (9x^2y^2 + 48x^3y^3) dx dy; \quad D: x = 1, y = \sqrt{x}, y = -x^2$

3. $\iint_D y^2 \sin \frac{xy}{2} dx dy; \quad D: x = 0, y = \sqrt{\pi}, y = \frac{x}{2}$

4. $x^2 + 4x + y^2 = 0, x^2 + 8x + y^2 = 0, y = 0, y = -x$

5. $D: x^2 + y^2 = 1, x^2 + y^2 = 4, y = \frac{x}{\sqrt{3}}, y = 0 (x \geq 0, y \geq 0), \mu = \frac{x + y}{x^2 + y^2}$

6. $\int_L (x^2 - 2xy) dx + (y^2 - 2xy) dy, L: y = x^2, M(-1; 1), N(1; 1)$

7. $\int_L (-y dx + x dy),$

$L: x = 2 \cos t, y = 2 \sin t, (x \geq 0, y \geq 0), M(2; 0), N(0; 2)$

8. $y' = 2 + y^2, M(1; 2)$

9. $y' \sin x = y \ln y$

10. $y' + y \operatorname{ctg} x = \cos x, y\left(\frac{\pi}{2}\right) = \frac{1}{2}$

11. $2xy'y'' = (y')^2 - 1$

12. $y''' + 25y' = 0$

13. $y'' + 2y' + y = 3x + 5$

14. $y'' - 3y' + 2y = 10 \sin x, y(0) = 1, y'(0) = 0$

Вариант 3.

$$1. \int_0^1 dy \int_0^{\sqrt{y}} f(x, y) dx + \int_1^{\sqrt{2}} dy \int_0^{\sqrt{2-y^2}} f(x, y) dx$$

$$2. \iint_D (36x^2y^2 - 96x^3y^3) dx dy; \quad D: x = 1, y = \sqrt[3]{x}, y = -x^3$$

$$3. \iint_D y \cos xy dx dy; \quad D: y = \frac{\pi}{2}, y = \pi, x = 1, x = 2$$

$$4. y^2 + 6y + x^2 = 0, y^2 + 8y + x^2 = 0, x = 0, y = -\sqrt{3}x$$

$$5. D: x^2 + y^2 = 9, x^2 + y^2 = 25, y = -x, y = 0 (x \leq 0, y \geq 0), \mu = \frac{y - 2x}{x^2 + y^2}$$

$$6. \int_L y dx + \frac{x}{y} dy, L: y = e^{-x}, M(-1; e), N(0; 1)$$

$$7. \int_L (x + 2y) dx + (x - y) dy,$$

$$L: x = 4 \cos t, y = 4 \sin t, (x \geq 0, y \geq 0), M(4; 0), N(0; 4)$$

$$8. y' = (y - 1)x, M(1; 3/2)$$

$$9. y' \sin x - y \cos x = 0$$

$$10. y' - 4xy = 2x e^{x^2}, y(0) = 1$$

$$11. y'' = \frac{y'}{x} + 1$$

$$12. y'''' - 7y'' = 0$$

$$13. y'' - 9y = 5xe^{2x}$$

$$14. y'' + 4y' + 5y = 25x, y(0) = 2, y'(0) = 0$$

Вариант 4.

1. $\int_0^{3/2} dy \int_{2y^2}^{y+3} f(x, y) dx$

2. $\iint_D (18x^2y^2 + 32x^3y^3) dx dy; \quad D: x = 1, y = x^3, y = -\sqrt[3]{x}$

3. $\iint_D y^2 e^{-\frac{xy}{4}} dx dy; \quad D: x = 0, y = 2, y = x$

4. $x^2 - 2x + y^2 = 0, x^2 - 4x + y^2 = 0, y = 0, y = x$

5. $D: x^2 + y^2 = 9, x^2 + y^2 = 16, y = \sqrt{3}x, y = 0 (x \geq 0, y \geq 0), \mu = \frac{2x + 5y}{x^2 + y^2}$

6. $\int_L \frac{y^2 + 1}{y} dx - \frac{x}{y^2} dy, L: \text{отрезок } MN, M(1; 2), N(2; 4)$

7. $\int_L (x^2 - y) dx + (x - y^2) dy,$

$L: x = 5 \cos t, y = 5 \sin t, (x \leq 0, y \leq 0), M(-5; 0), N(0; -5)$

8. $y' = 3 + y^2, M(1; 2)$

9. $(5 + y^2) + y' y (1 - x^2) = 0$

10. $y' - 4xy = 4x^3 e^{2x^2}, y(0) = 0$

11. $y'' \operatorname{ctg} x + 2y' = 0$

12. $y''' - 3y'' - 4y' = 0$

13. $y'' + 2y' + 5y = 17 \sin 2x$

14. $y'' - 6y' + 9y = 9x^2 - 3x - 4, y(0) = 1, y'(0) = 5$

Вариант 5.

$$1. \int_0^1 dy \int_0^{\sqrt{y}} f(x, y) dx + \int_1^e dy \int_{\ln y}^1 f(x, y) dx$$

$$2. \iint_D (27x^2y^2 + 48x^3y^3) dx dy; \quad D: x = 1, y = x^2, y = -\sqrt[3]{x}$$

$$3. \iint_D y \sin xy dx dy; \quad D: y = \frac{\pi}{2}, y = \pi, x = 1, x = 2$$

$$4. y^2 - 8y + x^2 = 0, y^2 - 10y + x^2 = 0, y = \frac{x}{\sqrt{3}}, y = \sqrt{3}x$$

$$5. D: x^2 + y^2 = 4, x^2 + y^2 = 36, x = 0, y = x (x \leq 0, y \leq 0), \mu = \frac{-x - y}{x^2 + y^2}$$

$$6. \int_L (xy - x^2) dx + x dy, \quad L: y = 2x^2, \quad M(0; 0), \quad N(1; 2)$$

$$7. \int_L (x + y) dx + 2x dy,$$

$$L: x = 2 \cos t, y = 2 \sin t, (x \geq 0), \quad M(0; -2), \quad N(0; 2)$$

$$8. y'(x^2 + 2) = y, \quad M(2; 2)$$

$$9. y \ln y + x y' = 0$$

$$10. y' - 3x^2 y = x^2 e^{x^3}, \quad y(0) = 0$$

$$11. x y'' - 2y' = -\frac{2}{x^2}$$

$$12. y''' - 3y'' - 4y' = 0$$

$$13. y'' - 2y' + y = (2x + 5) e^{2x}$$

$$14. y'' - 4y' + 13y = 26x + 5, \quad y(0) = 1, \quad y'(0) = 1$$

Вариант 6.

$$1. \int_0^4 dy \int_{3y/4}^{\sqrt{25-y^2}} f(x, y) dx$$

$$2. \iint_D (18x^2y^2 + 32x^3y^3) dx dy; \quad D: x = 1, y = \sqrt[3]{x}, y = -x^2$$

$$3. \iint_D y^2 \cos \frac{xy}{2} dx dy; \quad D: x = 0, y = \sqrt{\frac{\pi}{2}}, y = \frac{x}{2}$$

$$4. x^2 - 4x + y^2 = 0, x^2 - 8x + y^2 = 0, y = 0, y = \sqrt{3}x$$

$$5. D: x^2 + y^2 = 1, x^2 + y^2 = 16, x = 0, y = \frac{x}{\sqrt{3}} (x \geq 0, y \geq 0), \mu = \frac{x + 2y}{x^2 + y^2}$$

$$6. \int_L \frac{y}{x} dx + x dy, \quad L: y = \ln x, M(1; 0), N(e; 1)$$

$$7. \int_L (2xy - y) dx + (x^2 + x) dy,$$

$$L: x = 3 \cos t, y = 3 \sin t, (y \leq 0), M(-3; 0), N(0; -3)$$

$$8. y' = y - x, M(9/2; 1)$$

$$9. (1 - x^2) y' + xy^2 + x = 0$$

$$10. y' - \frac{y}{x} = -\frac{2}{x^2}, y(1) = 1$$

$$11. x y'' + 2y' = 0$$

$$12. y''' + 5y'' - 14y' = 0$$

$$13. y'' + y = x^2 + 6$$

$$14. y'' - 5y' - 6y = e^x (-10x - 3), y(0) = 0, y'(0) = 8$$

Вариант 7.

1. $\int_0^1 dx \int_{1-x^2}^1 f(x, y) dy + \int_1^e dx \int_{\ln x}^1 f(x, y) dy$

2. $\iint_D (18x^2y^2 + 32x^3y^3) dx dy; \quad D: x = 1, y = x^3, y = -\sqrt{x}$

3. $\iint_D 4ye^{2xy} dx dy; \quad D: y = \ln 3, y = \ln 4, x = \frac{1}{2}, x = 1$

4. $y^2 + 4y + x^2 = 0, y^2 + 6y + x^2 = 0, x = 0, y = -\frac{x}{\sqrt{3}}$

5. $D: x^2 + y^2 = 25, x^2 + y^2 = 36, y = -x, y = 0 (x \geq 0, y \leq 0), \mu = \frac{x - y}{x^2 + y^2}$

6. $\int_L (x^2 + y) dx - (y^2 + x) dy, L: \text{отрезок } MN, M(1; 2), N(3; 5)$

7. $\int_L xy dx + 2y dy,$

$L: x = \cos t, y = \sin t, (x \leq 0), M(0; 1), N(0; -1)$

8. $y' = xy, M(0; -1)$

9. $\sqrt{4 - x^2} y' + x (y^2 + 1) = 0$

10. $y' - \frac{2y}{x+1} = (x+1)^3, y(0) = \frac{1}{2}$

11. $(1 + \sin x) y'' = y' \cos x$

12. $y'''' - 81y = 0$

13. $y'' - 4y' + 3y = -4x e^x$

14. $y'' + 6y' + 9y = 25 e^{2x}, y(0) = 3, y'(0) = 2$

Вариант 8.

1. $\int_0^1 dy \int_0^{y^2+1} f(x, y) dx$

2. $\iint_D (27x^2y^2 + 48x^3y^3) dx dy; \quad D: x = 1, y = \sqrt{x}, y = -x^3$

3. $\iint_D 4y^2 \sin xy dx dy; \quad D: x = 0, y = \sqrt{\frac{\pi}{2}}, y = x$

4. $x^2 + 2x + y^2 = 0, x^2 + 10x + y^2 = 0, y = 0, y = \sqrt{3}x$

5. $D: x^2 + y^2 = 4, x^2 + y^2 = 25, x = 0, y = \frac{x}{\sqrt{3}} (x \leq 0, y \leq 0), \mu = \frac{-2x - 3y}{x^2 + y^2}$

6. $\int_L (xy - x) dx + \frac{x^2}{2} dy, L: y = 2\sqrt{x}, M(0; 0), N(1; 2)$

7. $\int_L (x^2 + y^2) dx + (x^2 - y^2) dy,$

$L: x = 6 \cos t, y = 6 \sin t, (y \geq 0), M(6; 0), N(-6; 0)$

8. $yy' = -\frac{x}{2}, M(4; 2)$

9. $y' y \sqrt{\frac{1-x^2}{1-y^2}} + 1 = 0$

10. $y' + \frac{y}{2x} = x, y(1) = 0$

11. $x^3 y'' + x^2 y' = 1$

12. $y''' - 9y'' + 8y' = 0$

13. $y'' - y' - 2y = (1 - 2x) e^x$

14. $y'' - 2y' + y = 16e^x, y(0) = 1, y'(0) = 2$

Вариант 9.

$$1. \int_0^1 dy \int_0^{\sqrt[3]{y}} f(x, y) dx + \int_1^2 dy \int_0^{2-y} f(x, y) dx$$

$$2. \iint_D (4xy + 3x^2y^2) dx dy; \quad D: x = 1, y = x^2, y = -\sqrt{x}$$

$$3. \iint_D y \cos 2xy dx dy; \quad D: y = \frac{\pi}{2}, y = \pi, x = \frac{1}{2}, x = 1$$

$$4. y^2 - 6y + x^2 = 0, y^2 - 10y + x^2 = 0, y = x, x = 0$$

$$5. D: x^2 + y^2 = 9, x^2 + y^2 = 36, y = -\sqrt{3}x, y = 0 (x \leq 0, y \geq 0), \mu = \frac{2y - 4x}{x^2 + y^2}$$

$$6. \int_L \frac{y}{x^2 + y^2} dx + \frac{x}{x^2 + y^2} dy, \quad L: \text{отрезок } MN, M(1; 2), N(3; 6)$$

$$7. \int_L (x + y\sqrt{x^2 + y^2}) dx + x dy,$$

$$L: x = \cos t, y = \sin t, (y \leq 0), M(-1; 0), N(0; -1)$$

$$8. y' = x + 2y, M(3; 0)$$

$$9. \sqrt{4 + x^2} dx - 4y dy = x^2 y dy$$

$$10. y' - \frac{y}{x} = x^2, y(1) = 0$$

$$11. x^5 y'' + x^4 y' = 9$$

$$12. y''' - 6y'' + 9y' = 0$$

$$13. y'' + 6y' + 13y = 75 \cos 2x$$

$$14. y'' + y = 4e^x, y(0) = 4, y'(0) = -3$$

Вариант 10.

1. $\int_0^4 dx \int_{\sqrt{x}}^{2\sqrt{x}} f(x, y) dy$

2. $\iint_D (12xy + 9x^2y^2) dx dy; \quad D: x = 1, y = \sqrt{x}, y = -x^2$

3. $\iint_D y^2 e^{-\frac{xy}{8}} dx dy; \quad D: x = 0, y = 2, y = \frac{x}{2}$

4. $x^2 - 2x + y^2 = 0, x^2 - 4x + y^2 = 0, y = \frac{x}{\sqrt{3}}, y = \sqrt{3}x$

5. $D: x^2 + y^2 = 1, x^2 + y^2 = 9, y = -\sqrt{3}x, y = 0 (x \geq 0, y \leq 0), \mu = \frac{x - y}{x^2 + y^2}$

6. $\int_L (x^2 - 2y) dx + (y^2 - 2x) dy, L: \text{отрезок } MN, M(-4; 0), N(0; 2)$

7. $\int_L x^2 y dx - xy^2 dy,$

$L: x = 2 \cos t, y = 2 \sin t, (x \leq 0, y \geq 0), M(0; 2), N(-2; 0)$

8. $3yy' = x, M(-3; -2)$

9. $x \sqrt{1 + y^2} + y y' (1 + x^2) = 0$

10. $y' - \frac{y}{x} = x \sin x, y \left(\frac{\pi}{2} \right) = \frac{\pi}{2}$

11. $x^2 y'' + x y' = 1$

12. $y''' - 2y'' - 8y' = 0$

13. $y'' + 2y' + y = 2 - 3x^2$

14. $y'' + 81y = 162 e^{9x}, y(0) = 0, y'(0) = 9$

Вариант 11.

1. $\int_0^1 dy \int_{-\sqrt{y}}^0 f(x, y) dx + \int_1^2 dy \int_{-\sqrt{2-y}}^0 f(x, y) dx$

2. $\iint_D (8xy + 9x^2y^2) dx dy; \quad D: x = 1, y = \sqrt[3]{x}, y = -x^3$

3. $\iint_D 12y \sin 2xy dx dy; \quad D: y = \frac{\pi}{4}, y = \frac{\pi}{2}, x = 2, x = 3$

4. $y^2 + 2y + x^2 = 0, y^2 + 4y + x^2 = 0, x = 0, y = x$

5. $D: x^2 + y^2 = 1, x^2 + y^2 = 36, x = 0, y = -x (x \geq 0, y \leq 0), \mu = \frac{2x - y}{x^2 + y^2}$

6. $\int_L \frac{y}{x} dx + (x^3 + 1) dy, L: y = \ln x, M(1; 0), N(e; 1)$

7. $\int_L (x^2 + \sqrt{x^2 + y^2}) dx + (y - \sqrt{x^2 + y^2}) dy,$

$L: x = 4 \cos t, y = 4 \sin t, (x \geq 0, y \leq 0), M(0; -4), N(4; 0)$

8. $x^2 - y^2 + 2xyy' = 0, M(-2; 1)$ 9. $y(1 - \ln y) + xy' = 0$

10. $xy' + y = \ln x, y(1) = 1$ 11. $2xy'' = y'$

12. $y''' - 6y'' + 12y' - 8y = 0$ 13. $y'' + y' - 6y = 10e^{2x}$

14. $y'' + y = 1, y\left(\frac{\pi}{2}\right) = 0, y'\left(\frac{\pi}{2}\right) = 0$

Вариант 12.

1. $\int_0^1 dx \int_{2x+1}^{4-x^2} f(x, y) dy$

2. $\iint_D (24xy + 18x^2y^2) dx dy; \quad D: x = 1, y = x^3, y = -\sqrt[3]{x}$

3. $\iint_D y^2 \cos xy dx dy; \quad D: x = 0, y = \sqrt{\pi}, y = x$

4. $x^2 + 2x + y^2 = 0, x^2 + 6x + y^2 = 0, y = 0, y = x$

5. $D: x^2 + y^2 = 9, x^2 + y^2 = 25, x = 0, y = -\frac{x}{\sqrt{3}} (x \leq 0, y \geq 0), \mu = \frac{2y - x}{x^2 + y^2}$

6. $\int_L (2xy + y^2) dx - x dy, L: y = 2x^2, M(-1; 2), N(0; 0)$

7. $\int_L y^2 dx - x^2 dy,$

$L: x = 5 \cos t, y = 5 \sin t, (x \leq 0, y \leq 0), M(-5; 0), N(0; -5)$

8. $y' = y - x, M(2; 1)$

9. $2x + 2xy^2 + (2 - x^2) y' = 0$

10. $y' + \frac{y}{x} = 3x, y(1) = 1$

11. $xy'' + y' = x + 1$

12. $y''' + 2y'' - 24y' = 0$

13. $y'' + 3y' + 2y = (6x - 1) e^x$

14. $y'' + 9y = 18x + 9, y(0) = 0, y'(0) = 5$

Вариант 13.

$$1. \int_0^1 dy \int_{-y}^0 f(x, y) dx + \int_1^{\sqrt{2}} dy \int_{-\sqrt{2-y^2}}^0 f(x, y) dx$$

$$2. \iint_D (12xy + 27x^2y^2) dx dy; \quad D: x = 1, y = x^2, y = -\sqrt[3]{x}$$

$$3. \iint_D ye^{\frac{xy}{4}} dx dy; \quad D: y = \ln 2, y = \ln 3, x = 4, x = 8$$

$$4. y^2 - 4y + x^2 = 0, y^2 - 6y + x^2 = 0, y = \sqrt{3}x, x = 0$$

$$5. D: x^2 + y^2 = 16, x^2 + y^2 = 36, y = x, y = 0 (x \geq 0, y \geq 0), \mu = \frac{x + 6y}{x^2 + y^2}$$

$$6. \int_L 2xy dx - x^2 dy, L: y = 2x^2, M(0; 0), N(1; 2)$$

$$7. \int_L (-y dx + (2xy + x) dy),$$

$$L: x = 3 \cos t, y = 3 \sin t, (y \geq 0), M(3; 0), N(-3; 0)$$

$$8. y' = x^2 - y, M(0; 1)$$

$$9. 2x dx - y dy = y x^2 dy - x y^2 dx$$

$$10. y' + \frac{3y}{x} = \frac{2}{x^3}, y(1) = 1$$

$$11. y'' \operatorname{tg} x = y' + 1$$

$$12. y''' + 4y'' + 4y' = 0$$

$$13. y'' + 2y' - 3y = 30 \cos 3x$$

$$14. y'' - 2y' = 2e^x, y(0) = 0, y'(0) = 0$$

Вариант 14.

1. $\int_0^2 dx \int_{x^2/4}^{2\sqrt{x}} f(x, y) dy$

2. $\iint_D (8xy + 18x^2y^2) dx dy$; $D: x = 1, y = \sqrt[3]{x}, y = -x^2$

3. $\iint_D 4y^2 \sin 2xy dx dy$; $D: x = 0, y = \sqrt{2\pi}, y = 2x$

4. $x^2 - 2x + y^2 = 0, x^2 - 8x + y^2 = 0, y = \frac{x}{\sqrt{3}}, y = \sqrt{3}x$

5. $D: x^2 + y^2 = 4, x^2 + y^2 = 16, x = 0, y = \sqrt{3}x (x \leq 0, y \leq 0), \mu = \frac{-2y - 3x}{x^2 + y^2}$

6. $\int_L (x + y)^2 dx - (x^2 + y^2) dy, L: \text{отрезок } MN, M(0; 1), N(1; 0)$

7. $\int_L (x - y) dx + dy,$

$L: x = 2 \cos t, y = 2 \sin t, (y \leq 0), M(-2; 0), N(2; 0)$

8. $yy' = -2x, M(0; 5)$

9. $(1 + e^x) yy' = e^x$

10. $y' + \frac{y}{x} = e^x, y(1) = 0$

11. $xy'' + y' + x = 0$

12. $y''' + 3y'' - 4y' = 0$

13. $y'' - 3y' + 2y = -5e^x$

14. $y'' + y = -\sin(2x), y(\pi) = 1, y'(\pi) = 1$

Вариант 15.

$$1. \int_0^1 dy \int_0^{y^3} f(x, y) dx + \int_1^2 dy \int_0^{2-y} f(x, y) dx$$

$$2. \iint_D (24xy - 48x^3y^3) dx dy; \quad D: x = 1, y = x^2, y = -\sqrt{x}$$

$$3. \iint_D 2y \cos 2xy dx dy; \quad D: y = \frac{\pi}{4}, y = \frac{\pi}{2}, x = 1, x = 2$$

$$4. y^2 + 2y + x^2 = 0, y^2 + 6y + x^2 = 0, x = 0, y = \frac{x}{\sqrt{3}}$$

$$5. D: x^2 + y^2 = 25, x^2 + y^2 = 49, y = 0, y = -\sqrt{3}x (x \leq 0, y \geq 0), \mu = \frac{4y - x}{x^2 + y^2}$$

$$6. \int_L y^2 dx + y dy, \quad L: y = \sin x, \quad M(-\pi; 0), \quad N(0; 0)$$

$$7. \int_L y dx - x dy,$$

$$L: x = \sqrt{2} \cos t, y = \sqrt{2} \sin t, (x \geq 0), \quad M(0; -\sqrt{2}), \quad N(0; \sqrt{2})$$

$$8. y' = \frac{2x}{3y}, \quad M(1; 1)$$

$$9. \sqrt{5 + y^2} dx + 4(x^2 y + y) dy = 0$$

$$10. y' + \frac{2xy}{1 + x^2} = \frac{3x^2}{1 + x^2}, \quad y(0) = 0$$

$$11. y'' \operatorname{tg} x = y'$$

$$12. y''' - 9y'' + 8y' = 0$$

$$13. y'' + y' - 2y = 9e^x$$

$$14. y'' + y = 48 \cos 5x + 72 \sin 5x, \quad y(0) = 0, \quad y'(0) = 0$$

Вариант 16.

1. $\int_0^4 dx \int_{x/2+1}^{7-x} f(x, y) dy$

2. $\iint_D (6xy + 24x^3y^3) dx dy; \quad D: x = 1, y = \sqrt{x}, y = -x^2$

3. $\iint_D y^2 e^{-\frac{xy}{2}} dx dy; \quad D: x = 0, y = \sqrt{2}, y = x$

4. $x^2 + 2x + y^2 = 0, x^2 + 4x + y^2 = 0, y = 0, y = -x$

5. $D: x^2 + y^2 = 9, x^2 + y^2 = 16, x = 0, y = -\sqrt{3}x (x \leq 0, y \geq 0), \mu = \frac{2y - 5x}{x^2 + y^2}$

6. $\int_L 2y dx + (3x - y) dy, L: y = \sqrt{x}, M(1; 1), N(4; 2)$

7. $\int_L (-x dx + y dy),$

$L: x = 3 \cos t, y = 3 \sin t, (x \geq 0, y \geq 0), M(3; 0), N(0; 3)$

8. $yy' + x = 0, M(-2; -3)$

9. $(e^{2x} + 2) dy + y e^{2x} dx = 0$

10. $y' + \frac{y}{x} = \sin x, y(\pi) = 1$

11. $xy'' - y' + \frac{1}{x} = 0$

12. $y''' + 36y' = 0$

13. $y'' - 6y' + 9y = 4x e^x$

14. $y'' - 3y' + 2y = 24e^{-2x}, y(0) = -1, y'(0) = 4$

Вариант 17.

$$1. \int_{-2}^{-1} dy \int_0^{y+2} f(x, y) dx + \int_{-1}^0 dy \int_0^{\sqrt{-y}} f(x, y) dx$$

$$2. \iint_D (4xy + 16x^3y^3) dx dy; \quad D: x = 1, y = \sqrt[3]{x}, y = -x^3$$

$$3. \iint_D y \sin xy dx dy; \quad D: y = \pi, y = 2\pi, x = \frac{1}{2}, x = 1$$

$$4. y^2 - 2y + x^2 = 0, y^2 - 10y + x^2 = 0, y = \frac{x}{\sqrt{3}}, y = \sqrt{3}x$$

$$5. D: x^2 + y^2 = 4, x^2 + y^2 = 49, y = \sqrt{3}x, y = 0 (x \leq 0, y \leq 0), \mu = \frac{-2x - 4y}{x^2 + y^2}$$

$$6. \int_L (2xy^2 - 1)y dx - (3xy^2 + 5)x dy, \quad L: \text{отрезок } MN, M(0; 0), N(2; 4)$$

$$7. \int_L (x^2 - y) dx + (x + y^2) dy,$$

$$L: x = 2 \cos t, y = 2 \sin t, (y \geq 0), M(2; 0), N(-2; 0)$$

$$8. xy' = 2y, M(2; 3)$$

$$9. x dx - 3y dy = y x^2 dy - x y^2 dx$$

$$10. y' - \frac{y}{x+1} = e^x (x+1), y(0) = 1$$

$$11. y'' \operatorname{ctg} x = 2y'$$

$$12. y''' + 3y'' + 3y' + y = 0$$

$$13. y'' + 3y' + 2y = 12x^2 + 8x$$

$$14. y'' - 5y' + 4y = 3e^{4x}, y(0) = 0, y'(0) = 4$$

Вариант 18.

$$1. \int_0^3 dx \int_0^{\sqrt{4-x}} f(x, y) dy$$

$$2. \iint_D (4xy + 16x^3y^3) dx dy; \quad D: x = 1, y = x^3, y = -\sqrt[3]{x}$$

$$3. \iint_D y^2 \cos 2xy dx dy; \quad D: x = 0, y = \sqrt{\frac{\pi}{2}}, y = \frac{x}{2}$$

$$4. x^2 - 2x + y^2 = 0, x^2 - 6x + y^2 = 0, y = 0, y = \frac{x}{\sqrt{3}}$$

$$5. D: x^2 + y^2 = 1, x^2 + y^2 = 16, x = 0, y = \sqrt{3}x (x \geq 0, y \geq 0), \mu = \frac{x + 3y}{x^2 + y^2}$$

$$6. \int_L (x^2 + 4xy) dx + (2xy + y^2) dy, L: y = x^2, M(1; 1), N(2; 4)$$

$$7. \int_L (x + y) dx + (x - y) dy,$$

$$L: x = 4 \cos t, y = 4 \sin t, (x \leq 0, y \leq 0), M(-4; 0), N(0; -4)$$

$$8. 2(y + y') = x + 3, M(1; 1/2)$$

$$9. (x^2y + 9y) dy + \sqrt{2 + y^2} dx = 0$$

$$10. x y' + y = x^5, y(1) = 0$$

$$11. (1 + x^2) y'' + 2x y' = 2$$

$$12. y''' + 4y'' - 5y' = 0$$

$$13. y'' + 25y = 50 e^{5x}$$

$$14. y'' - y = 2x, y(0) = 0, y'(0) = 0$$

Вариант 19.

$$1. \int_0^1 dy \int_0^y f(x, y) dx + \int_1^{\sqrt{2}} dy \int_0^{\sqrt{2-y^2}} f(x, y) dx$$

$$2. \iint_D (44xy + 16x^3y^3) dx dy; \quad D: x = 1, y = x^2, y = -\sqrt[3]{x}$$

$$3. \iint_D 8ye^{4xy} dx dy; \quad D: y = \ln 3, y = \ln 4, x = \frac{1}{4}, x = \frac{1}{2}$$

$$4. y^2 + 4y + x^2 = 0, y^2 + 10y + x^2 = 0, x = 0, y = -x$$

$$5. D: x^2 + y^2 = 9, x^2 + y^2 = 49, y = 0, y = -\sqrt{3}x (x \leq 0, y \geq 0), \mu = \frac{3y - x}{x^2 + y^2}$$

$$6. \int_L \frac{y^2}{x} dx - x^2 dy, \quad L: y = \ln x, \quad M(1; 0), \quad N(e; 1)$$

$$7. \int_L (2x - y) dx + x dy,$$

$$L: x = 3 \cos t, y = 3 \sin t, (y \leq 0), \quad M(-3; 0), \quad N(3; 0)$$

$$8. yy' = -x, \quad M(2; 3)$$

$$9. x\sqrt{5+y^2} dx + y\sqrt{4+x^2} dy = 0$$

$$10. y' - y \operatorname{ctg} x = 2x \sin x, \quad y\left(\frac{\pi}{2}\right) = \frac{\pi^2}{4}$$

$$11. xy'' + y' = \frac{1}{\sqrt{x}}$$

$$12. y''' + y'' - 2y' = 0$$

$$13. y'' - 3y' + 2y = -5e^x$$

$$14. y'' - 64y = 128 \cos 8x, \quad y(0) = 0, \quad y'(0) = 0$$

Вариант 20.

1. $\int_0^1 dx \int_{-\sqrt{1-x^2}}^{1-x} f(x, y) dy$

2. $\iint_D (4xy + 176x^3y^3) dx dy; \quad D: x = 1, y = \sqrt[3]{x}, y = -x^3$

3. $\iint_D 3y^2 \sin \frac{xy}{2} dx dy; \quad D: x = 0, y = \sqrt{\frac{4\pi}{3}}, y = \frac{2}{3}x$

4. $x^2 + 2x + y^2 = 0, x^2 + 6x + y^2 = 0, y = 0, y = x$

5. $D: x^2 + y^2 = 1, x^2 + y^2 = 4, x = 0, y = \frac{x}{\sqrt{3}} (x \geq 0, y \geq 0), \mu = \frac{x + 2y}{x^2 + y^2}$

6. $\int_L \left(y - \frac{1}{y} \right) dx + \left(\frac{x}{y} - 2 \right) dy, L: y = \frac{1}{x}, M(1; 1), N \left(2; \frac{1}{2} \right)$

7. $\int_L (x + y) dx + (2x - y) dy,$

$L: x = 5 \cos t, y = 5 \sin t, (x \geq 0, y \leq 0), M(0; -5), N(5; 0)$

8. $3yy' = x, M(1; 1)$

9. $6x dx - y dy = yx^2 dy - 3xy^2 dx$

10. $y' - \frac{y}{x} = x^2, y(1) = 0$

11. $x^4 y'' + x^3 y' = 1$

12. $y''' + 6y'' + 5y' = 0$

13. $y'' + y = 2 \cos 7x - 3 \sin 7x$

14. $y'' + 3y' + 2y = 1 - 2x^2, y(0) = 0, y'(0) = 0$

Вариант 21.

$$1. \int_0^1 dx \int_0^{x^2} f(x, y) dy + \int_1^{\sqrt{2}} dx \int_0^{\sqrt{2-x^2}} f(x, y) dy$$

$$2. \iint_D (xy - 4x^3y^3) dx dy; \quad D: x = 1, y = x^3, y = -\sqrt{x}$$

$$3. \iint_D y \cos xy dx dy; \quad D: y = \pi, y = 3\pi, x = \frac{1}{2}, x = 1$$

$$4. y^2 - 2y + x^2 = 0, y^2 - 4y + x^2 = 0, y = \frac{x}{\sqrt{3}}, x = 0$$

$$5. D: x^2 + y^2 = 36, x^2 + y^2 = 49, y = 0, y = -x (x \leq 0, y \geq 0), \mu = \frac{4y - 2x}{x^2 + y^2}$$

$$6. \int_L (x^2 + y^2) dx + \frac{x^3}{y} dy, \quad L: y = e^{2x}, M(0; 1), N(1; e^2)$$

$$7. \int_L (x + y) dx + (x - y) dy,$$

$$L: x = 3 \cos t, y = 3 \sin t, (y \leq 0), M(-3; 0), N(3; 0)$$

$$8. y' = x + 2y, M(1; 2)$$

$$9. (2 - e^x) dy + 3e^x \operatorname{tg} y dx = 0$$

$$10. y' - y \operatorname{tg} x = 1, y(0) = 0$$

$$11. y'' x \ln x = y'$$

$$12. y''' + 6y'' + 9y' = 0$$

$$13. y'' + y = 16 \cos 3x - 24 \sin 3x$$

$$14. y'' + 6y' + 5y = 84e^{2x}, y(0) = -1, y'(0) = 1$$

Вариант 22.

1. $\int_0^{1/4} dy \int_y^{\sqrt{y}} f(x, y) dx$

2. $\iint_D (4xy + 176x^3y^3) dx dy; \quad D: x = 1, y = \sqrt{x}, y = -x^3$

3. $\iint_D y^2 e^{-\frac{xy}{2}} dx dy; \quad D: x = 0, y = 1, y = \frac{x}{2}$

4. $x^2 - 2x + y^2 = 0, x^2 - 4x + y^2 = 0, y = 0, y = \sqrt{3}x$

5. $D: x^2 + y^2 = 1, x^2 + y^2 = 9, y = \frac{x}{\sqrt{3}}, y = 0 (x \leq 0, y \leq 0), \mu = \frac{-2x - y}{x^2 + y^2}$

6. $\int_L \left(y + \frac{1}{y} \right) dx - \frac{x}{y^2} dy, L: y = x^3, M(1; 1), N(2; 8)$

7. $\int_L y^2 dx + xy dy,$

$L: x = 3 \cos t, y = 3 \sin t, (x \leq 0), M(0; 3), N(0; -3)$

8. $x^2 - y^2 + 2xyy' = 0, M(2; 1)$

9. $y' = (2y + 1) \operatorname{ctg} x$

10. $y' - \frac{3y}{x} = x, y(1) = 6$

11. $y'' - \frac{y'}{x(2 + \ln x)} = 2 + \ln x$

12. $y''' + y' = 0$

13. $y'' + 4y' + 4y = 8x^2 + 6$

14. $y'' - y' = 2x, y(0) = 0, y'(0) = 0$

Вариант 23.

1. $\int_0^2 dy \int_{y/2}^y f(x, y) dx + \int_2^4 dy \int_{y/2}^2 f(x, y) dx$

2. $\iint_D (9x^2y^2 + 25x^4y^4) dx dy; \quad D: x = 1, y = \sqrt{x}, y = -x^2$

3. $\iint_D y \sin 2xy dx dy; \quad D: y = \frac{\pi}{2}, y = \frac{3\pi}{2}, x = \frac{1}{2}, x = 3$

4. $y^2 + 6y + x^2 = 0, y^2 + 8y + x^2 = 0, x = 0, y = x$

5. $D: x^2 + y^2 = 1, x^2 + y^2 = 49, y = -\sqrt{3}x, y = 0 (x \leq 0, y \geq 0), \mu = \frac{3y - 2x}{x^2 + y^2}$

6. $\int_L 2xy dx + (x^2 + 2) dy, L: y = \frac{x^2}{4}, M(-2; 1), N(0; 0)$

7. $\int_L \left(-\frac{y}{x^2 + y^2} dx + \frac{x}{x^2 + y^2} dy \right),$

$L: x = 4 \cos t, y = 4 \sin t, (x \leq 0, y \geq 0), M(0; 4), N(-4; 0)$

8. $y' = x(y - 1), M(1; 1/2)$

9. $\sqrt{3 + y^2} + \sqrt{1 - x^2} y y' = 0$

10. $y' - \frac{y}{x} = \ln x, y(1) = 0$

11. $x^5 y'' + x^4 y' = 1$

12. $y''' - 3y'' + 2y' = 0$

13. $y'' + y' = 4x - 1$

14. $y'' - 2y' = e^x (3x - 1), y(0) = 2, y'(0) = 0$

Вариант 24.

1. $\int_0^1 dx \int_{x^2}^{2x^2+1} f(x, y) dy$

2. $\iint_D (54x^2y^2 + 150x^4y^4) dx dy; \quad D: x = 1, y = x^2, y = -\sqrt[3]{x}$

3. $\iint_D y^2 \cos xy dx dy; \quad D: x = 0, y = \sqrt{\pi}, y = 2x$

4. $x^2 + 4x + y^2 = 0, x^2 + 8x + y^2 = 0, y = 0, y = -x$

5. $D: x^2 + y^2 = 1, x^2 + y^2 = 25, x = 0, y = -\sqrt{3}x (x \geq 0, y \leq 0), \mu = \frac{x - 4y}{x^2 + y^2}$

6. $\int_L (3x^2y + 1) dx + (x^3 + 2) dy, L: y = 2\sqrt{x}, M(0; 0), N(1; 2)$

7. $\int_L x^3 dx - y^3 dy,$

$L: x = 2 \cos t, y = 2 \sin t, (x \geq 0), M(0; -2), N(0; 2)$

8. $y' = y - x^2, M(-3; 4)$

9. $x\sqrt{1 - y^2} dx + y\sqrt{1 - x^2} dy = 0$

10. $y' + y \cos x = \cos x e^{-\sin x}, y(0) = 1$

11. $xy'' + y' = 3x + 2$

12. $y'''' - 16y = 0$

13. $y'' - 2y' + y = e^{6x}$

14. $y'' - 4y' + 3y = 10 \cos x, y(0) = 1, y'(0) = 2$

Вариант 25.

$$1. \int_1^2 dy \int_{-\sqrt{y-1}}^{\sqrt{y-1}} f(x, y) dx + \int_2^5 dy \int_{-\sqrt{y-1}}^{3-y} f(x, y) dx$$

$$2. \iint_D (xy - 9x^5y^5) dx dy; \quad D: x = 1, y = \sqrt[3]{x}, y = -x^2$$

$$3. \iint_D 6ye^{\frac{xy}{3}} dx dy; \quad D: y = \ln 2, y = \ln 3, x = 3, x = 6$$

$$4. y^2 - 4y + x^2 = 0, y^2 - 8y + x^2 = 0, x = 0, y = \sqrt{3}x$$

$$5. D: x^2 + y^2 = 4, x^2 + y^2 = 16, y = 0, y = -x (x \geq 0, y \leq 0), \mu = \frac{3x - 4y}{x^2 + y^2}$$

$$6. \int_L (y^2 + x) dx + \frac{2x}{y} dy, \quad L: y = e^{3x}, M(0; 1), N(1; e^3)$$

$$7. \int_L xy dx + y^2 dy,$$

$$L: x = 4 \cos t, y = 4 \sin t, (x \leq 0, y \geq 0), M(0; 4), N(-4; 0)$$

$$8. xy' = 2y, M(1; 3)$$

$$9. y(5 + \ln y) + xy' = 0$$

$$10. y' - y \cos x = \cos^2 x e^{\sin x}, y(0) = 0$$

$$11. x^4 y'' + x^3 y' = 4$$

$$12. y'''' - 9y'' = 0$$

$$13. y'' + y' = x$$

$$14. y'' + 4y = e^{-2x}, y(0) = 0, y'(0) = 0$$